International Journal of Research in Advanced Engineering and Technology
www.newengineeringjournal.in
ISSN: 2455-0876
Received: 26-06-2022, Accepted: 14-07-2022, Published: 27-07-2022
Volume 8, Issue 2, 2022, Page No. 17-19

An analytical study on higher natural frequencies of stepped beam using spectral finite elements

Nitesh Kumar[1], Sandeep Pannu[2]
[1] Department of Civil Engineering (Structural Engineering), Matu Ram Institute of Engineering and Management, Rohtak, Haryana, India
[2] Assistant Professor, Department of Civil Engineering, Matu Ram Institute of Engineering and Management, Rohtak, Haryana, India

Abstract
In engineering, the dynamic behaviour of a structure is critical, and precise prediction of the structure's dynamic properties is essential. Structural dynamics makes heavy use of the finite element method (FEM). As long as the structure's wavelength is large relative to the mesh size, a finite-element model may offer realistic dynamic properties. In fact, as the frequency of the calculations rises, the finite element solutions become progressively erroneous. Refining the mesh may increase accuracy, but the cost might be exorbitant. Traditional finite element (mass and stiffness) matrices are often constructed using presumed frequency-independent polynomial shape functions. Vibration shapes of structures change with frequency, hence a correct FEM needs a mesh of finite elements (or elements) to represent the structure in real-time. Alternatively, the subdivision may not be required if the form functions are frequency dependent.

Keywords: structural, dynamics, subdivision, finite, element, method

Introduction

The dynamic features of a structure's dynamic behaviour must be precisely predicted through engineering. FEM is widely used in structural dynamics. Finite element models may be accurate if structural wavelengths are large compared to mesh sizes. Solutions based on finite elements grow more erroneous as their frequency rises.

Because of the short incidence pulse duration (microseconds) and high frequency content of the wave propagation problem (kHz). Higher-level modes are activated when the structure receives such a pulse. The wavelength of a high-frequency wave is short. The typical finite element method requires a thin mesh to match wavelengths in order to capture higher order patterns. This adds to the size of the system. SEA may be the best answer to these issues. The governing equation is first discretized by means of the Fourier transform in order to perform SEA. This simplifies the controlling PDE to a set of ODEs with constant coefficients and frequency as a parameter for 1D waveguides. It is easier to solve the ODEs compared to the PDEs. Controlling ODEs in the frequency domain are interpolated using SEA. Accurate solution yields a mass distribution and dynamic stiffness matrix with perfect precision and accuracy. A single element can handle any length beam if there is no discontinuity in the signal. The system is substantially more compact when compared to more standard FEM systems. To begin, determine the dynamic stiffness of the system (frequency response function). As a result, the container is completely full. An IEFT is used to retrieve the response's time history.

There are two alternative approaches that may be used in order to find a solution to the governing differential equations in the time domain. Methods such as numerical integration and mode analysis are examples of time-domain methodologies for vibration analysis that fall within the first group. There are other options accessible in the frequency domain. This procedure makes use of "frequency-domain" technologies that are referred to as SEM. To solve differential equations using SEM, it is possible to superimpose an endless number of wave modes, each of which has a distinct frequency. The CFT for the solutions looks like this. For the purpose of reproducing the temporal histories of solutions, IFTs are performed on an infinite number of frequency-domain spectrum components. The Continuous Fourier transform is only able to deal with fundamental functions, and the IFT presents a significant challenge in the vast majority of real-world scenarios. The DFT is used much more often than the CFT.

If the frequency of interest is high enough, the fast Fourier transform (FFT) approach may be used to analyse the highest number of spectral components possible. Spectrum representations may be used to characterise dynamic responses for structural components as an alternative to the basic harmonic solutions, which are thought to define the dynamic stiffness matrix. This may be done in place of the basic harmonic solutions. This is because spectrum representations have become far more precise in recent years. This method of spectrum analysis, which is also known as the dynamic stiffness matrix, may be used to construct a dynamic stiffness matrix. [Case in point:] [Case in point:] This matrix may be obtained in its entirety for each frequency component in its own right. On the other hand, FEM utilises the spectral element as the finite structural element that corresponds to the matrix of spectral elements. Spectral elements are used in the matrix. This occurs as a result of the presence of a finite element. The forward fast Fourier transform (FFT) approach is used whenever there is a need for a spectrum representation of any time-domain external elements. It is possible to formulate an equation for the global spectral system matrix using the spectral elements of the overall structure. To rebuild the dynamic responses, it is essential to utilise the IFFT for each frequency component of the spectral DOF matrix equation. This is required in order to do so.

Fast Fourier Transforms (FFTs)

In signal processing, fourier transformations are essential. To define, characterise, and analyse discretized temporal signals, digital computations use Discrete Fourier Transforms (DFT). On the other hand, direct implementation of DFT is quite inefficient in terms of computation. There are several DFT methods, but Cooley-Tukey is the simplest and most often used. Derivatives of Fourier transforms (DFTs) are called Fast Fourier Transforms because of their speed.

SEM

In order to solve the governing differential equations in the time domain, two techniques may be utilised. Examples of time-domain approaches for vibration analysis in the first category include numerical integration and mode analysis. Frequency-domain methods are also available. "frequency-domain" methods known as SEM are used in this process. An infinite number of wave modes with varied frequencies may be superimposed to solve differential equations in SEM. This is the CFT of the solutions. IFTs are done on an unlimited number of frequency-domain spectrum components to reproduce the temporal histories of solutions. Continuous Fourier transform can only handle basic functions, and the IFT is a severe problem in most real cases. The DFT is more often used than the CFT.

As long as the frequency of interest is high enough, the greatest amount of spectral components may be analysed using the FFT method. Instead of the basic harmonic solutions, which are considered to describe the dynamic stiffness matrix, spectrum representations may be utilised to characterise dynamic responses for structural components. This is due to the fact that spectrum representations are now more accurate. A dynamic stiffness matrix may be generated using this approach of spectrum analysis, which is also known as the dynamic stiffness matrix. It's possible to get this matrix for each frequency component separately. FEM, on the other hand, uses the spectral element as the finite structural element that corresponds to the matrix of spectral elements A finite element is the reason behind this. If a spectrum representation of any time-domain external factors is required, the forward FFT technique is used. The spectral elements may be used to create a global spectral system matrix equation for the whole structure. For each frequency component of the spectral DOF matrix equation, it is necessary to use the IFFT to reconstruct the dynamic responses.

Methodology

The same spectral elements formulation that is used for a uniform beam is utilised to a stepped beam in a stepped beam when assessing two elements that have been obtained with different cross-sectional areas. This ensures that the results of the assessment are accurate.

The following is one possible way to express the element stiffness matrix for the beam:

$$\begin{Bmatrix} V_1 \\ m_1 \\ V_2 \\ m_2 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} \hat{v}_1 \\ \hat{\phi}_1 \\ \hat{v}_2 \\ \hat{\phi}_2 \end{Bmatrix}$$

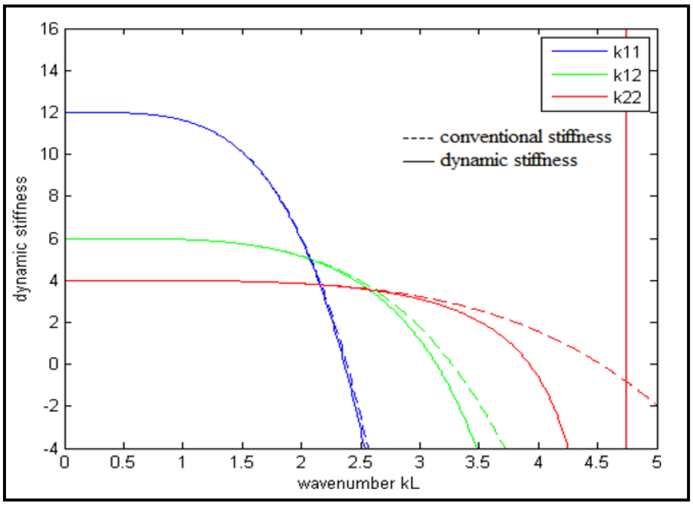

Fig 1: Comparison of conventional and spectral stiffness of a beam

The present dynamic stiffness is compared with the dynamic stiffness of previous designs in figure 1, which may be found here. It is evident that both exhibit the same limiting behaviour at low frequencies, since "they agree to order 2(or (kL)4) terms when the constant mass matrix is used." At kL 2 and beyond, the conventional form functions in a repetitive manner, while the current form shows a large number of zeros. In order for the conventional element to have these zeros, they need to be put together.

Result

In the process of researching a "Euler-Bournelli Fixed-Free Beam," spectral element formulation was used in order to determine the natural frequencies of the beam's first ten modes and to make a comparison with the exact answer that was found. Due to the fact that the first three natural frequencies are identical, Table 1 reveals that SEM is capable of quickly capturing higher modes of frequencies. SFEM is capable of recovering all of the natural "frequencies of the first 10 modes of a stepped beam" using just two elements, which are used in the process of extracting the natural frequencies of a stepped beam.

Table 1: Euler-Bournelli Stepped Beam (Natural Frequencies by Dynamic Stiffness

Mode no	"I_2/I_1=5"		"I_2/I_1=10"		"I_2/I_1=20"		"I_2/I_1=40"	
	Present	[34]	Present	[34]	Present	[34]	Present	[34]
1.	28.1689	25.1850	24.5559	28.5459	21.4715	23.4755	23.9669	25.1807
2.	38.0878	79.0579	87.8659	88.8560	81.2591	84.2552	98.4531	
3.	88.4831		85.4631		81.4131		96.1584	92.1284
4.	138.828		156.567	156.627	114.910	175.985	209.458	205.458
5.	192.555	182.582	158.183		181.254		255.093	
6.	255.652	265.663	296.681		241.6191		276.681	
7.	276.671		258.341	269.652	257.015	265.055	283.584	253.525
8.	379.057	349.080	399.855	368.866	454.211	454.252	484.488	454.458
9.	367.887		488.646		463.653		463.683	
10.	473.713		487.623		531.648		687.383	647.344

Conclusion

In this article, a method (SEM) that is conceptually analogous to the finite element method (FEM) is discussed. Using this method, problems that include several connected beams and rods may be solved in an uncomplicated and uncomplicated manner. In contrast to conventional finite elements, the length of a spectral element is irrelevant to the accuracy with which it is specified; this is due to the fact that spectral elements are defined independently of one another. Therefore, the length of the element is established according to the structural connections as well as any discontinuities. Because of this, the overall number of equations that need to be solved has been cut down by a significant margin. The SEM is capable of precisely calculating the natural frequencies of both the lower and upper modes even though it is only comprised of a maximum of two components. This is because of how efficiently the approach makes use of its components. The natural frequencies of uniform and stepped beams up to ten numbers of modes have been obtained by using two different numbers of spectral finite elements for a range of boundary conditions. This has allowed for up to ten numbers of modes to be calculated.

References

1. Abdelhmid MK, McConnell KG. A spectral analysis method for non-stationary field measurements, 1986.
2. Abramovich H. Shear deformation and rotatory inertia effects of vibrating composites beams. Composite structure,1992:20:165-173.
3. Abramovich H, Livshits A. Free vibration of non-symmetric cross-ply laminated composite beams. Journal of Sound and Vibration,1994:176:597-612.
4. Banerjee JR. Free vibration of axially loaded composite Timoshenko beams using the dynamic stiffness matrix method. Computers and Structures,1998:69:197-208.
5. Banerjee JR. Free vibration of centrifugally stiffened uniform and tapered beams using the dynamic stiffness method. Journal of Sound and Vibration,2000:233:857-875.
6. Banerjee JR. Dynamic stiffness formulation for structural elements: A general approach Computers and Structures,1997:63(1):101-103.
7. Banerjee JR, Guo S, Howson WP. Exact dynamic stiffness matrix of a bending- torsion coupled beam including warping. Computers & structures.,1996:59:613-621.
8. Banerjee JR, Fisher SA. Coupled bending torsional dynamic stiffness matrix for axially loaded beam element. International Journal numer.Meth.Engng,1992:33:739-751.
9. Banerjee JR, Williams FW. Clamped-Clamped natural frequencies of a bending- torsion coupled beam. Journal of Sound and Vibration.1994:176:301-306.