International Journal of Research in Advanced Engineering and Technology
www.newengineeringjournal.in
ISSN: 2455-0876
Received: 20-06-2022, Accepted: 05-07-2022, Published: 22-07-2022
Volume 8, Issue 2, 2022, Page No. 6-12

Effect of hall current on MHD of a Casson fluid in a vertical porous plate with thermal radiation

P Valsamy[1], G Arunagiri[2]
[1] Department of Engineering Mathematics, FEAT Annamalai University, Annamalai Nagar, Chidambaram, Tamil Nadu, India
[2] Department of Mathematics, Annamalai University, Annamalai Nagar, Chidambaram, Tamil Nadu, India

Abstract
This paper aims to described steady two-dimensional laminar MHD free convection flowing, Casson fluid flow past a stretching vertical plate surface in the presence of Hall effects, free convection, thermal radiation, viscous dissipation, heat generation or absorption and rate of chemical reaction are studied., The governing equations are transformed to the non-linear differential equation using the similarity approaches. Perturbation approach is applied to solve the resulting equations. Outcomes of the velocity field, temperature profile and concentration field are discussed through graphs.

Keywords: MHD, Casson fluid, hall current, viscous dissipation, porous medium

Introduction
Analyzes the combined effect of the free convective heat and mass transfer on the steady two-dimensional boundary layer flow over a vertical plate by taking heat generation/absorption. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. In astronomy, geophysics, and engineering, the influence of a Casson fluid running freely across through an endless vertical plate has an in-depth range of essential technical packages. The main objective of this chapter is to take into account its Soret and Dufour effect also on the a stable natural convection flow layer flow of an electrically conductive and Casson fluid by moving vertically down a low-heat-resistant sheet in the porous media, Heat source and sinks. The governing equations are similarity converted, and the resulting dimensionless equations, Graphs are drawn for velocity distribution, temperature profile and concentration. A mathematical investigation on the peristaltic transport of a Casson fluid has been investigated by Mernone et.al [1]. Attia [2] considered the hydrodynamic impulsively lid-driven flow and heat transfer of a Casson fluid. Pramanik [3] considered Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation. Kabir [4]) discussed the unsteady Casson fluid flow through parallel plates with hall current, Joule Heating and viscous dissipation. Raju [5] has made a heat and mass transfer in Magnetohydrodynamic Casson fluid over an exponentially permeable stretching surface. Pushpalatha [6] heat and mass transfer in unsteady MHD Casson fluid flow with convective boundary conditions. Sarojamma [7]. Imran [8] examined MHD Casson fluid flow, heat and mass transfer in a vertical channel with stretching walls. General solutions of convective flows of mhd Casson fluid with slip and radiative heat transfer at the boundary. Sandeep [9] discussed effect of radiation and chemical reaction on transient MHD free convective flow over a vertical plate through porous media. Ibrahim [10] considered effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction.
This work analyzes the flow of effect hall current of a through the vertical porous plate with outcome of thermal radiation, viscous dissipation and mass transfer. Non-linear System of differential equations are derived by perturbation approach. Results of various parameters displayed in graphs.

Result
Mathematical formulation
We have considered an steady two-dimensional laminar MHD free convection flowing, Casson fluid flow past a stretching vertical plate surface in the presence of Hall effects, free convection, thermal radiation, viscous dissipation, heat generation or absorption and rate of chemical reaction. A transverse magnetic fieldB0 is also applied, making hall current effects under the account. The governing boundary layer equality in the flow field is developed on the tracking hypothesis. Stagnation point $U_x(x) = bx$ and $U_e(x) = ax$ describes stretched velocity and free stream velocity over the direction of x-axis, where b > 0 and a > 0 are rate of stretching.The temperature and concentration of the wall surface are denoted by T_W and C_W. At the a distance from the surface, the temperature and concentration of the fluid T_∞ and C_∞, As well hall current are taken into consideration for a modified generalised Ohm's law The isotropic rheological equation for incompressible Casson fluid is denoted by

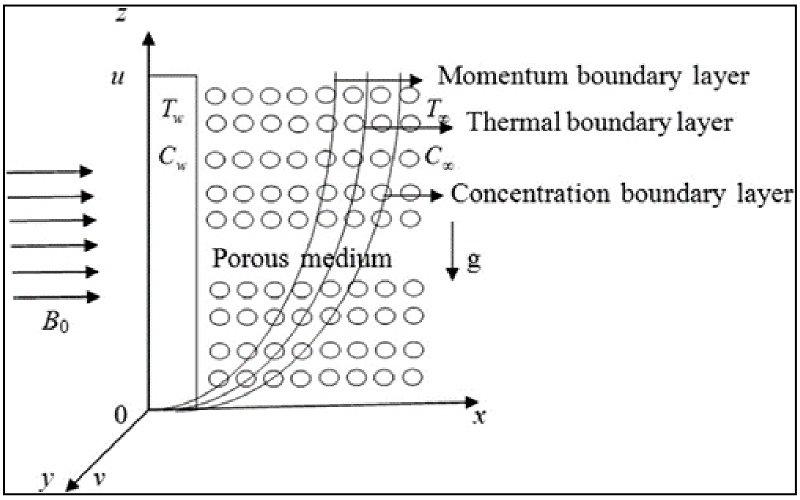

Fig 1: the physical model

The modified generalized Ohm's law, which takes into account Hall current described by

$$\boldsymbol{J} = \frac{\sigma}{1+m^2}\left(E + V \times B - \frac{1}{en_e} J \times B\right) \tag{1}$$

From V are the velocity vector and E -are the intensity vector about the electric field, B- magnetic induction vectore, electric current density vector- j, the hall parameter $m = \left(\frac{\sigma B_0}{en_e}\right)$, electric conductivity - σ, e - charge of electron, ne - number density of electron, hall current produces a force in z -course which generates a move flow velocity on this direction, and for this reason the float becomes three dimensional. the following is an example of a velocity field. We further assume that under certain assumptions, the electrical field E = 0 falls to

$$j_x = \frac{\sigma \mu_e H_0}{1+m^2}(mu - w) \tag{2}$$

$$j_z = \frac{\sigma \mu_e H_0}{1+m^2}(u + mw) \tag{3}$$

For an incompressible Casson fluid, the isotropic rheological equation is denoted as, [Mustafa, et.al., [16]]

$$\tau_{ij} = \begin{cases} \left(\frac{\mu_B + P_y}{\sqrt{2\pi}}\right) 2e_{ij}, & \pi > \pi_c \\ \left(\frac{\mu_B + P_y}{\sqrt{2\pi_c}}\right) 2e_{ij}, & \pi < \pi_c \end{cases} \tag{4}$$

where μ_B, P_y, $\pi = e_{ij}\, e_{ij}$ and π_c are defined the plastic dynamic viscosity, yield stress, rate of product deformation of $(i, j)th$ component, critical value.

Governing equations of the present model are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho(1+m^2)}(u + mw) + g\beta_T(T - T_\infty) + \frac{\nu}{K^*}u \tag{6}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2}{\rho(1+m^2)}(mu - w) \tag{7}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}+\frac{Q}{\rho c_p}(T-T_\infty) \tag{8}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D_m\frac{\partial^2 C}{\partial y^2}-K(C-C_\infty) \tag{9}$$

The boundary conditions for the governing equations are defined as,

$$\left.\begin{aligned} u=U_w(x)=Cx, v=0, w=0, T=T_w, C=C_w \;\; at\; y=0 \\ u\to 0, w\to 0, T\to T_\infty, T\to T_\infty \; at\; y=\infty \end{aligned}\right\} \tag{10}$$

where u, v and w are the velocity components in the x-, y- and z-directions, respectively, β denotes Casson fluid parameter, β_T are thermal expansion coefficient, σ denotes electrical conductivity, ρ refers to the density of fluid, g defines gravitational acceleration, υ denotes kinematic viscosity, k thermal conductivity of fluid, T and C are the fluid temperature and concentration, The temperature and concentration of the wall surface are denoted by T_W and C_W. At the a distance from the surface, the temperature and concentration of the fluid T_∞ and C_∞, K^* denotes permeability of the porous medium, the source term Q > 0 and sink term Q_0 < 0, υ are kinematic viscosity, D_m is mass diffusivity.
The following non-dimensional similarity transformation are introduced by,
q_r denotes approximated Rosseland raditive heat flux is represented as Turkyilmazoglu, [17]

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{11}$$

Here σ^* and k^* indicates constant of Stefan's and absorption of mean coefficient. Extending the T^4 in expansion of Taylor series over T_∞ and omitting the higher terms we get,

$$T^4\approx 4T_\infty{}^3T-3T_\infty{}^4 \tag{12}$$

Substituting the eqn (12) in eqn (11) we get,

$$q_r=-\frac{4\sigma^*\partial(4T_\infty{}^3T-3T_\infty{}^4)}{3k^*\partial y^*}$$

$$q_r=-\frac{16T_\infty{}^3}{3k^*}\;\frac{\partial T}{\partial y} \tag{13}$$

Applying equation (13) in equation (8) we get,

$$u\frac{\partial T}{\partial x}+\;v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\;=\frac{k}{\rho c_p}\left(1+\frac{16\sigma^*T_\infty{}^3}{3kk^*}\right)\frac{\partial^2 T}{\partial y^2}$$

$$\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}+\frac{Q}{\rho c_p}(T-T_\infty) \tag{14}$$

$$\left.\begin{aligned} u=Cxf'(\eta),\; v=-\sqrt{cv}f(\eta),\; \psi=Cxh(\eta), w=\sqrt{cv}f(\eta), \\ \eta=y\sqrt{\frac{C}{v}},\; \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\; \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \end{aligned}\right\} \tag{15}$$

Where, the similarity parameter η and $f(\eta)$, $\theta(\eta)$ are the non-dimensionless variables.
The above dimensionless parameter denoted as

$$M=\frac{\sigma B_0^2}{\rho C}, K=\frac{v}{k^*c}, Gr=\frac{g\beta_T(T_w-T_\infty)}{C^2x},\; R=\frac{16\sigma^*T_\infty{}^3}{3kk^*}$$

$$Pr=\frac{\mu c_p}{k},\;\; Du=\frac{D_m k_T}{c_s c_p}\frac{C_w-C_w}{T_w-T_\infty}, Q=\frac{Q_0}{c_\rho c_p},\; Sc=\frac{\gamma}{D_m}, \beta\frac{\gamma}{c}.$$

where *M, K, Gr, R, Pr, Du, Q, Sc and* γ are defined as magnetic field parameter, porosity parameter, Grashof number, Radiation number Prandtl number, Dufour number, heat generatin/absorption term, Schmidth number and chemical reaction term

By using the stream function,the continuity equation (5) is automatically met,

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x},\qquad w=0 \tag{16),}$$

Now substituting Eq. (16) in Eqs. (6), (7), (8) and Eq. (9), the governing equations become

$$\left(1+\frac{1}{\beta}\right)f_0'''(\eta)-[f_0'(\eta)]^2+f(\eta)f''(\eta)-\frac{M}{1+m^2}[f'(\eta)+Mh(\eta)]+ Gr\theta(\eta)+Kf'(\eta)=0 \tag{12}$$

$$\left(1+\frac{1}{\beta}\right)f''^{(\eta)}-f'^{(\eta)}h(\eta)+h'^{(\eta)}f(\eta)$$

$$-\frac{\mathrm{M}}{1+\mathrm{m}^2}[\mathrm{m}f'(\eta)-\mathrm{h}(\eta)]=0 \tag{13}$$

$$(1+R)\theta_0''+Prf(\eta)\theta'(\eta)+PrDu\varphi''(\eta)+PrQ\theta(\eta)=0 \tag{14}$$

$$\varphi''(\eta)+\mathrm{Sc}f(\eta)\varphi'(\eta)-\mathrm{Sc}\beta\varphi=0 \tag{15}$$

the modified boundary conditions are:

$$\left.\begin{array}{c} f'(\eta)=1, f(\eta)=h(\eta)=0, \theta(\eta)=1, \varphi(\eta)=1\ at\ \eta=0 \\ f'(\eta)=0, h(\eta)=0, \theta(\eta)\to 0, \varphi(\eta)=0\ at\ \eta=\infty \end{array}\right\} \tag{16}$$

Method of Solution

The modified flow equations are resolved by perturbation technique

$$\left.\begin{array}{c} f(\eta)=f_0(\eta)+\epsilon f_1(\eta)+\epsilon^2 f_2(\eta)+\cdots \\ h(\eta)=h_0(\eta)+\epsilon h_1(\eta)+\epsilon^2 h_2(\eta)+\cdots \\ \theta(\eta)=\theta_0(\eta)+\epsilon\theta_1(\eta)+\epsilon^2\theta_2(\eta)+\cdots \\ \varphi(\eta)=\varphi_0(\eta)+\epsilon\varphi_1(\eta)+\epsilon^2\varphi_2(\eta)+\cdots \end{array}\right\} \tag{17}$$

Substituting the eqn. (17) into eqns. (12) to (15), omitting the higher order terms. we get,

Base part

$$\left(1+\frac{1}{\beta}\right)f_0'''(\eta)-[f_0'(\eta)]^2+f_0(\eta)f_0''(\eta)-\frac{M}{1+m^2}f_0'(\eta)+Kf_0'(\eta) -\frac{M}{1+m^2}mh_0(\eta)+Gr\varphi_0(\eta)=0 \tag{18}$$

$$\left(1+\frac{1}{\beta}\right)h_0''(\eta)-f_0'(\eta)h_0(\eta)+h_0'(\eta)f_0(\eta)-\frac{\mathrm{M}}{1+\mathrm{m}^2}\mathrm{m}f_0'(\eta)+\frac{\mathrm{M}}{1+\mathrm{m}^2}\mathrm{h}_0(\eta)=0 \tag{19}$$

$$(1+R)\theta_0''+Prf_0\theta_0'+PrDu\varphi''+PrQ\theta_0=0 \tag{20}$$

$$\varphi_0''+\mathrm{Sc}f\varphi'-\mathrm{Sc}\beta\varphi=0 \tag{21}$$

Perturbed part

$$(1+\tfrac{1}{\beta})f_0'''(\eta)-2f_0'(\eta)f_1'(\eta)+f_1(\eta)f_0''(\eta)f_0'(\eta)f_1''-\tfrac{M}{1+m^2}f_1'(\eta)+Kf_1'(\eta)$$

$$-\frac{M}{1+m^2}mh_1(\eta)+Gr\varphi_1(\eta)=0 \qquad (22)$$

$$\left(1+\frac{1}{\beta}\right)h_1''(\eta)-f_0'(\eta)h_1(\eta)-h_1'(\eta)f_0(\eta)+h_1'(\eta)f_0(\eta)-\frac{M}{1+m^2}mf_1'(\eta)+\frac{M}{1+m^2}h_1(\eta)=0 \qquad (23)$$

$$(1+R)\theta_1''+Prf_0\theta_1'+Prf_1\varphi_0''+PrDu\varphi_1''+PrQ\theta_1=0 \qquad (24)$$

$$\varphi_1''+Scf_0\varphi_1'+Scf_1\varphi_0'-Sc\,\varphi_1'\beta\varphi=0 \qquad (25)$$

Base part boundary condition

$$\left.\begin{matrix} f_0'=1, f_0=0, h_0=0, \theta_0=1, \varphi_0=1 \ at\ \eta=0 \\ f_0{}^1=0, h_0=0, \theta_0=0, \varphi_0=0 \ at\ \eta=\infty \end{matrix}\right\} \qquad (26)$$

Perturbed part boundary condition:

$$\left.\begin{matrix} f_1'=1, f_1=0, h_1=0, \theta_1=0, \varphi_1=0 \ at\ \eta=0 \\ f_1'=0, h_1=0, \theta_1=0, \varphi_1=0 \ at\ \eta=\infty \end{matrix}\right\} \qquad (27)$$

The above base and perturbed part equations are solved subject to the boundary condition (26) and (27). Graphs are drawn for velocity distribution, temperature profile and concentration distribution employing by Mathematica software.

Result and Discussion

The effects of various parameters on velocity profiles in the boundary layer are depicted in Figure (2-4). The figure (2) and (3) observed that axial field decreases with increases of Casson fluids and magnetic parameter. Figure (4) hall parameter m is axial field shown in figure. It is observed that the axial field decreases with increasing the hall parameter m. Figure (4) display the result of radiation parameter over temperature distribution that represents $\theta(\eta)$, reduced with enhancing cases of R. Figure (13) indicates that temperature distribution is decrease with increasing the heat generation /absorption parameter (Q). Figure (14) it is observed that concentration traveling at constant Su and γ

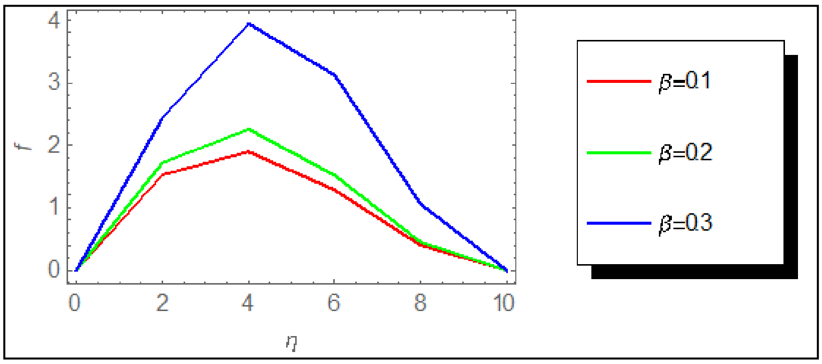

Fig 2: outcome of Casson fluid parameter β on velocity field f (η)

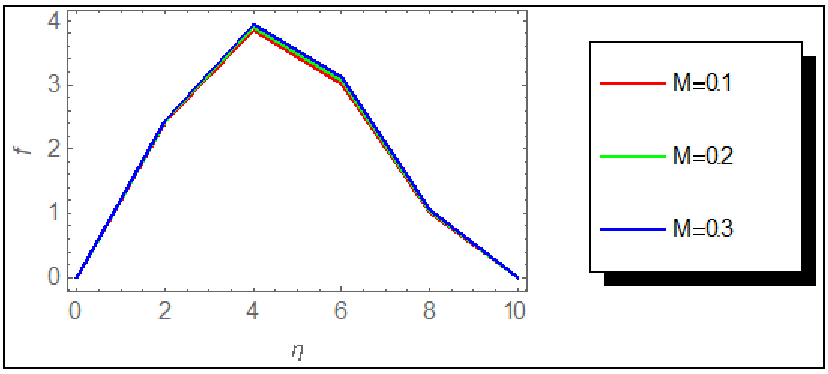

Fig 3: variation of M on velocity profile

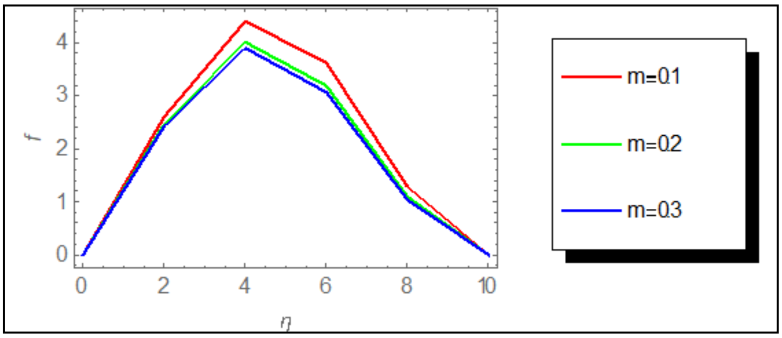

Fig 4: Variation of M on velocity profile

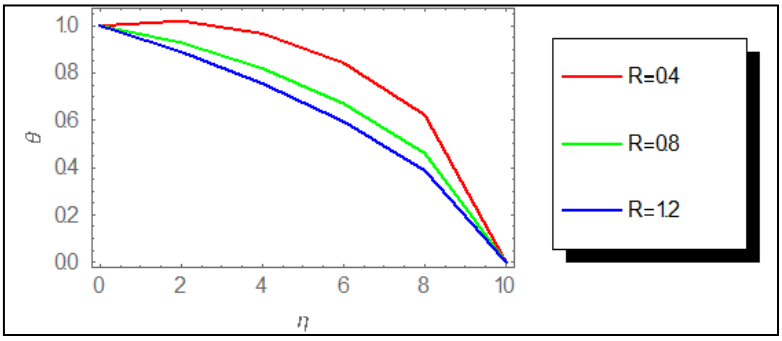

Fig 5: variation of R on temperature profile

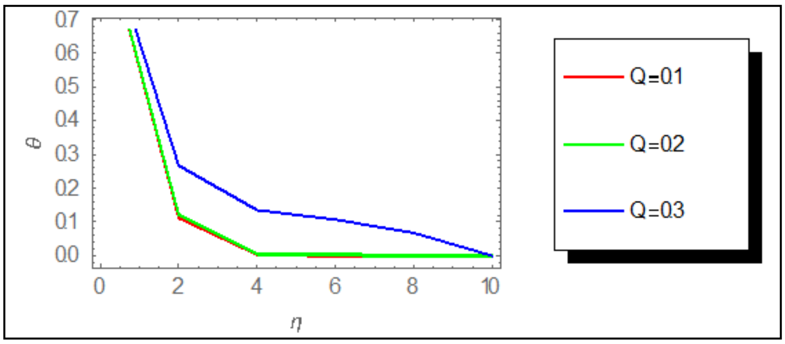

Fig 6: variation of Q on temperature profile

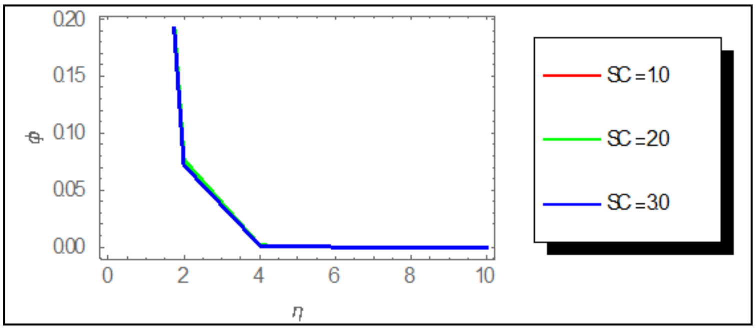

Fig 7: outcome of schmidth number Sc on concentration distribution $\varphi(\eta)$

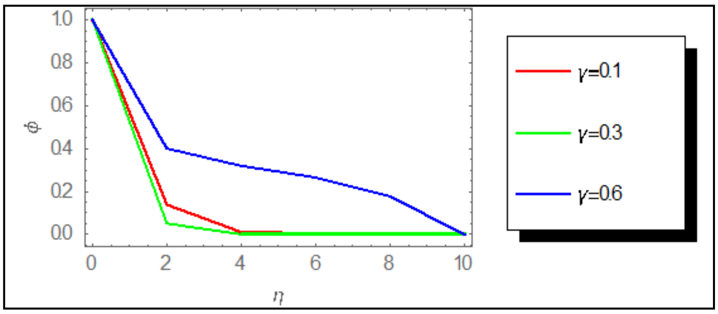

Fig 8: impact of chemical reaction term on concentration distribution $\varphi(\eta)$

Conclusion

A steady two-dimensional laminar MHD free convection flowing, its flow over a Casson fluid throughout some semi-endless vertically transfer the flat plate together in porous medium will investigate. The flow of effect hall current of a through the vertical porous plate with outcome of thermal radiation, viscous dissipation and mass transfer. Non-linear System of differential equations are derived by perturbation approach. Results of various parameters displayed in graphs.

References

1. Sayed-Ahmed ME, Attia HA and Ewis KM. "Time Dependent Pressure Gradient Effect on Unsteady MHD Couette Flow and Heat Transfer of a Casson Fluid", Canadian Journal of Physics,2011:3(1):38-49.
2. Mukhopadhyay S, De PR, Bhattacharyya K and Layek GC. "Casson Fluid Flow over an Unsteady Stretching Surface", Ain Shams Engineering Journal,2013:4(4):933-938.
3. Pramanik S. "Casson Fluid Flow and Heat Transfer Past an Exponentially Porous Stretching Surface in Presence of Thermal Radiation", Ain Shams Engineering Journal,2014:5(1)205-212.
4. Kabir MF and Alam MM. "The Unsteady Casson Fluid Flow through Parallel Plates with Hall Current, Joule Heating and Viscous Dissipation", Amse Journal,2015:84(1):1-22.
5. Raju CSK, Sandeep N, Sugunamma V, Babu MJ and Reddy JR, "Heat and Mass Transfer in Magnetohydrodynamic Casson Fluid over an Exponentially Permeable Stretching Surface", Engineering Science and Technology, an International Journal, 2016:19(1):45-52.
6. Pushpalatha K, Sugunamma V *, Ramana Reddy JV and Sandeep N. Heat and Mass Transfer in Unsteady MHD Casson Fluid Flow with Convective Boundary Conditions, International Journal of Advanced Science and Technology,2016:91:19-38.
7. Sarojamma G, Vasundhara B, Vendabai K, MHD Casson Fluid Flow, Heat and Mass Transfer in a Vertical Channel with Stretching Walls, International Journal of Scientific and Innovative Mathematical Research (IJSIMR),2014:2(10):800-810.
8. Imran MA, ∗ Shah NA, Rafique K, Sohail A, & Sana Ejaz, General solutions of convective flows of mhd Casson fluid with slip and radiative heat transfer at the boundary, Computational Thermal Sciences, 2017:9(1):1-11.
9. Sandeep N, Vijaya Bhaskar Reddy A, Sugunamma V, "Effect of radiation and chemical reaction on transient MHD free convective flow over a vertical plate through porous media", Chemical and Process Engineering Research,2012:2:1-9.
10. Ibrahim FS, Elaiw AM, Bakr AA, "Effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction", Communications in Nonlinear Science and Numerical Simulation,2008:13:1050-1066.